

### **Question to James Annett:**

We have a question about something in your book. Your expression for the real part of the conductivity (Eq. (3.39)) derived from the London Equation differs by a factor of 2 from other treatments (such as Tinkham, Eq. (2.44a)). This may be related to the integral Eq. (3.37) extending to minus infinity (footnote 4 of Ferrell and Tinkham Phys. Rev. 109, 1398 (1956)), or to some more subtle issue. Are you able to clarify this issue for us?

### **James Annett response:**

Nice to hear from you, and thanks for using my book in your course.

There are a few mistakes in the book, which I'm gradually finding. But looking at this one it seems to be a difference in definition as much as anything. I'll have to think about the Tinkham version as to whether it is better or not compared to mine. If so I'll correct the book for the next edition.

As you suggest, I think the difference is simply from the integration, whether it is 0 to infinity or minus infinity to infinity. We agree exactly that the conductivity is Lorentzian centred at  $\omega=0$ , as in my 3.36 and Tinkham's 2.43. The integrated area of the Lorentzian is  $\pi$  from  $-\infty$  to  $\infty$  and  $\pi/2$  from 0 to  $\infty$ . Taking the limit  $\tau^{-1} \rightarrow 0$  the Lorentzian becomes a single peak centred at  $\omega=0$ , which I have written as a single delta function with weight  $\pi$ . In Tinkham's version there is one delta function with weight  $\pi/2$  containing all the spectral weight from  $\omega=0$  to  $\infty$ . But there must also be another delta function infinitesimally below  $\omega=0$ , containing all the spectral weight from  $-\infty$  to 0. These two delta functions at infinitesimal positive and negative frequencies are obviously mathematically equivalent to a single one with weight  $\pi$ , but I can't see now if there are advantages of one picture over the other.

Perhaps as a real microwave impedance expert you could let me know which version makes more sense to you? I suppose that when you measure impedance you do it over real positive frequencies, and then use Kramers Kronig relations written as integrals over the real positive frequencies to go from the real to the imaginary parts. In this case it is the spectral weight over positive frequencies that is important, and I guess the Tinkham version would be more useful. If you were using my version to obtain  $\sigma_2$  from  $\sigma_1$  you would have to note that the  $\omega=0$

delta function has only half its weight on the positive omega axis, and so the contribution from the delta function would be halved. What is important is that the real/imaginary pair of functions obey Kramers Kronig, which I think Tinkham's 2.44 a and b do integrated over positive frequencies, and my pair 3.41 do integrated over both positive and negative frequencies, as in my Fig 3.11.